A Parameterization of Collisions in the Boltzmann Equation by a Rotation Matrix and Boltzmann Collision Integral in Discrete Models of Gas Mixtures

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I. INTRODUCTION

Applications of the group theory constructions remains not broadly used yet in the kinetic theory, as it deserves to be[1,2]. In the paper, the new opportunities and advantages that the parameterization of two-particle collisions by a matrix from the group of rotations can provide are shown. Usually, to construct the collision integral in the Boltzmann kinetic equation a collision of two particles is determined by setting a direction ($n, n^2 = 1$) of the relative velocity vector of the particles after the collision[3]:

$$v' = \frac{mv + u + |v - u|n}{1 + m}; \quad u' = \frac{mv + u - m|v - u|n}{1 + m}; \quad \text{where } m = \frac{m_1}{m_2}, \quad \mathbf{v} = v - u;$$ (1)

In the paper we offer to construct the collision integral using a parameterization of a scattering by a rotation matrix $\widehat{R} \in O_3$ which is determined by Euler's angles φ, θ, ψ [4].

In this case, the transformation of the velocities due to a collision in contrast to case (1), becomes a linear one:

$$v' = \frac{mv + u + \widehat{R}(v - u)}{1 + m}, \quad u' = \frac{mv + u - m\widehat{R}(v - u)}{1 + m};$$ (2)

and is representing by scattering matrix $\widehat{S}$ a partitioned matrix (2×2 cells). The size of each cell obviously is (3×3):

$$\xi' = \widehat{S}\xi, \quad \xi = \begin{pmatrix} v \\ u \end{pmatrix}, \quad \widehat{S} = \begin{pmatrix} \dfrac{m + \widehat{R}}{1 + m} & \dfrac{1 - \widehat{R}}{1 + m} \\ \dfrac{m(1 - \widehat{R})}{1 + m} & \dfrac{1 + m\widehat{R}}{1 + m} \end{pmatrix};$$ (3)

Also in contrast to (1), that linear transformation due to $\left| \det \widehat{S} \right| = 1$ provides the equality of velocity volumes

$$dvdu = dv'du'$$ (4).

The matrixes $\widehat{S}(\widehat{R}), \widehat{R} \in O_3$ constitute a group: $\widehat{S}(\widehat{R}_1) \cdot \widehat{S}(\widehat{R}_2) = \widehat{S}(\widehat{R}_1 \cdot \widehat{R}_2)$, $\widehat{S}^{-1}(\widehat{R}) = \widehat{S}(\widehat{R}^{-1})$.

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14. ABSTRACT

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II. THE COLLISION INTEGRAL

To write the collision integral the integration over relative velocity directions should be replaced $\int \dfrac{d\Omega_n}{4\pi} \rightarrow \int \dfrac{d\widehat{R}}{16\pi^2}$ with integration on the invariant ($d\widehat{R} = d\widehat{R}_0 \widehat{R} = d\widehat{R}\widehat{R}_0 = d\widehat{R}^{-1}$) measure[4] over the group O_3: $d\widehat{R} = d\psi d\varphi \sin\theta d\theta, \quad 0 \le \psi, \theta, \varphi < 2\pi$, $\int d\widehat{R} = 16\pi^2$. As a result, we get the collision integral written in an explicitly invariant and simple form:

$$I(f, \Psi) = I(F) = \int b\left(v, \widehat{R}\right)\left[f(v')\Psi(u') - f(v)\Psi(u)\right]\frac{d\widehat{R}}{4\pi}du = \int b\left(v, \widehat{R}\right)\left[F\left(\widehat{S}(\widehat{R})\xi\right) - F(\xi)\right]\frac{d\widehat{R}}{4\pi}du, (5)$$

where $b\left(v, \widehat{R}\right) = v\sigma_\theta\left(v, \mu\left(\widehat{R}\right)\right)$ is a scattering indicatrix, $\mu\left(\widehat{R}\right) = \dfrac{v \cdot \widehat{R}v}{v^2}$ — cosine of the scattering angle, $F(\xi) = f \circ \Psi(\xi) = f(v)\Psi(u)$ is a two-particle velocity distribution function. Taking into consideration that $\delta(v' - v_0)\delta(u' - u_0) = \delta(\widehat{S}\xi - \xi_0) = \delta\left(\xi - \widehat{S}(\widehat{R}^{-1})\xi_0\right)$ we have decomposition formula for δ-functions:

$$I\left(\delta(v - v_0), \delta(u - u_0)\right) = I\left(\delta(\xi - \xi_0)\right) = \int \frac{d\widehat{R}}{4\pi} b\left(v_0, \widehat{R}\right)\left[\delta(v - v_0') - \delta(v - v_0)\right], \qquad (6)$$

And similar for Maxwellians:

$$I\left(f_M(v - v_0), \Psi_M(u - u_0)\right) = \int \frac{d\widehat{R}}{4\pi}\left[b_T\left(v, u_0', \widehat{R}\right)f_M(v - v_0') - b_T\left(v, u_0, \widehat{R}\right)f_M(v - v_0)\right], \qquad (7)$$

where $b_T\left(v, u_0, \widehat{R}\right) = \int b\left(v, \widehat{R}\right)\Psi_M(u - u_0)du$.

At the zero temperature limit we obtain the asymptotic equality similar to property (6):

$$I\left(f_M(v - v_0)\psi_M(u - u_0)\right)_{T \to 0} = \int \frac{d\widehat{R}}{4\pi}b\left(v_0, \widehat{R}\right)\left[f_M(v - v_0') - f_M(v - v_0)\right] + O(T), \qquad (8)$$

For Maxwell's molecules (and asymptotically in the limit $T \to 0$ in the general case) those two decompositions coincide. In the case of the Maxwell molecules the indicatrix b does not depend on a velocity and consequently equality (8) is satisfied at all temperatures and is a corollary of the invariance of the collision operator with constant collision frequency with respect to Gauss transformation[1]:

$$e^{\frac{kT}{2m_1}\nabla_v^2}I(f, \Psi) = I\left(e^{\frac{kT}{2m_1}\nabla_v^2}f, e^{\frac{kT}{2m_2}\nabla_u^2}\Psi\right), \qquad (9)$$

where

$$e^{\frac{kT}{2m}\nabla_v^2}f(v) = \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}}\int dv' e^{-\frac{m(v - v')^2}{2kT}}f(v'), \quad \nabla_v^2 = \left(\frac{\partial}{\partial v}\right)^2, \nabla_u^2 = \left(\frac{\partial}{\partial u}\right)^2.$$

III. THE COLLISION INTEGRAL FOR DISCRETE MODELS

The form of collision operator obtained above is very useful in an application to the discrete models[5-9]. An explicit expression for the collision integral suitable for the discrete models can be constructed, if we replace in collision integral (5) the averaging over rotations from group of rotations O_3 with an averaging over its discrete subgroup consisting of finite number K elements ($\int \frac{d\hat{R}}{16\pi^2} \rightarrow \frac{1}{K}\sum_{k=1}^{K}\hat{R}_k$):

$$(10)$$

$$I_d(f,\Psi) = \frac{4\pi}{K}\int du \sum_{k=1}^{K} b(\mathrm{v},\hat{R}_k)[f(v')\Psi(u') - f(v)\Psi(u)] = \frac{4\pi}{K}\int du \sum_{k=1}^{K} b(\mathrm{v},\hat{R}_k)[F(\hat{S}(\hat{R}_k)\xi) - F(\xi)]$$

The expressions given by formulae (6) and (8) in this case are reduced to the finite sums:

$$I_d(\delta(v-v_0)\delta(u-u_0)) = \frac{4\pi}{K}\sum_{k=1}^{K} b(\mathrm{v}_0,\hat{R}_k)[\delta(v-v_0') - \delta(v-v_0)] , \tag{11}$$

and

$$I_d(f_M(v-v_0)\psi_M(u-u_0))_{T\to0} = \frac{4\pi}{K}\sum_{k=1}^{K} b(\mathrm{v}_0,\hat{R}_k)[f_M(v-v_0') - f_M(v-v_0)] + O(T). \tag{12}$$

Formulae (11) and (12) give an explicit representation of the collision terms in the set of equations for discrete gas models, which can be obtained from the initial Boltzmann kinetic equations after a substitution of distribution functions in them in a form of an expansion on delta-functions or on Maxwell velocity clusters.

IV. SCATTERING VARIABLES AND AN INVARIANT ENSEMBLE OF PAIRS OF DISCRETE VELOCITIES.

A state of a pair of colliding particles in the kinetic theory is usually characterized by particle velocities v and u. The state also can be uniquely characterized by other variables that are linear combinations of the velocities v and u and which transform in collisions much simpler. It is convenient to choose variables p and w as such scattering variables. The variable w is a center of masses velocity, and p is a momentum of the first particle in a center of masses frame of reference. Formulae of transformation from a variables v, u to variables p, w and back look as follows:

$$\begin{pmatrix} p \\ w \end{pmatrix} = \hat{A}\cdot\begin{pmatrix} v \\ u \end{pmatrix}, \quad \hat{A} = \frac{m_1 m_2}{m_1 + m_2}\begin{pmatrix} 1 & -1 \\ m_2^{-1} & m_1^{-1} \end{pmatrix}; \qquad \begin{pmatrix} v \\ u \end{pmatrix} = \hat{A}^{-1}\cdot\begin{pmatrix} p \\ w \end{pmatrix}, \quad \hat{A}^{-1} = \begin{pmatrix} m_1^{-1} & 1 \\ -m_2^{-1} & 1 \end{pmatrix}. \tag{13}$$

The matrix of scattering in the variables p, w becomes cell-diagonal. In fact, a velocity of a center of masses after collision does not change ($w' = w$) and the momentum p is rotating by a rotation matrix $\hat{R}$, $p' = \hat{R}p$. Consequently, the matrix of scattering in these variables has a form:

$$\hat{S}_A = \hat{A}\hat{S}\hat{A}^{-1} = \begin{pmatrix} \hat{R} & 0 \\ 0 & 1 \end{pmatrix}; \tag{14}$$

If we will consider a model of gas in which not all possible scatterings in a pair of colliding particles happen, but only related to rotation matrixes $\hat{R} \in O_3(K) \subset O_3$ from some discrete subgroup $O_3(K)$ containing K elements, it is possible to construct an ensemble of pairs of discrete velocities, that transforms into itself by the collision transformations. There are several ways to construct such ensembles. We consider purely lattice way. In this case the velocity of a center of masses is given by an integer linear combination of the basis vectors of a lattice w_1, w_2, w_3, and the momentum of the first particle is given by a linear superposition of the vectors $m_0 w_1$, $m_0 w_2$, $m_0 w_3$ with integer coefficients. A set of six integer numbers $l_1, l_2, l_3, k_1, k_2, k_3$ will be coordinates of velocity pairs:

$$p(l_1, l_2, l_3) = l_1 m_0 w_1 + l_2 m_0 w_2 + l_3 m_0 w_3,$$
$$w(k_1, k_2, k_3) = k_1 w_1 + k_2 w_2 + k_3 w_3. \tag{15}$$

For discrete velocities in accordance to (13) one will have:

$$v_\alpha = v_\alpha(l_1, l_2, l_3, k_1, k_2, k_3) = \left(\frac{m_0}{m_\alpha} l_1 + k_1 \right) w_1 + \left(\frac{m_0}{m_\alpha} l_2 + k_2 \right) w_2 + \left(\frac{m_0}{m_\alpha} l_3 + k_3 \right) w_3 \in L_\alpha;$$

$$u_\beta = v_\beta(-l_1, -l_2, -l_3, k_1, k_2, k_3) \in L_\beta; \quad l_1, l_2, l_3, k_1, k_2, k_3 = 0, \pm 1, \pm 2, \ldots; \tag{16}$$

$$v'_\alpha = \left(\frac{m_0}{m_\alpha} l_1 \hat{R} + k_1 \right) w_1 + \left(\frac{m_0}{m_\alpha} l_2 \hat{R} + k_2 \right) w_2 + \left(\frac{m_0}{m_\alpha} l_3 \hat{R} + k_3 \right) w_3, \quad u'_{0\beta} = v'_\beta(-l_1, -l_2, -l_3, k_1, k_2, k_3);$$

The vectors w_1, w_2, w_3 are basic vectors of the Bravais lattice invariant regarding a discrete subgroup of rotations (the point groups),

$$\hat{R} w_i = \sum_k w_k T_{ki}(\hat{R}), \tag{17}$$

where $T_{ki}(\hat{R})$ are unimodular matrixes with integer matrix elements. In the theory of crystals[10,11] known that there are 14 types of the Bravais lattices and 7 point symmetry groups of these lattices: $S_2, C_{2h}, D_{2h}, D_{3h}, D_{4h}, D_{6h}, O_h$ corresponding to 7 syngonies (triclinic, monoclinic, rhombic, trigonal, tetragonal, hexagonal, cubic). These 7 groups have 32 point subgroups. The only group not having a preferential direction and consequently the most appropriate one for our purposes is the O_h group. That is the group of symmetry of a cube (isomorphic to the group of symmetry of an octahedron) is consisting of 48 elements. Its subgroups without a preferential direction are: the group of proper rotations of a cube $O(24)$, the group of symmetry of a tetrahedron $T_d(24)$, the group of proper rotations of a tetrahedron $T(12)$ and the $T_h(24)$ group of proper rotations of a tetrahedron with an inversion added. The number of elements in these groups is indicated in brackets. For plane Bravais lattices there are only 10 point groups of symmetry: C_1, C_2, C_3, C_4, C_6 that are the groups of rotations around axes of the 1-st, 2-nd, 3-rd, 4-th and 6-th order (angles of rotations in the groups are divisible by either $\frac{\pi}{2}$, or $\frac{\pi}{3}$) and the groups D_1, D_2, D_3, D_4, D_6 that are the groups of rotations C_n with a reflection in an axis lying in a plane of the lattice. For a one-dimensional lattice, obviously, there are only two groups. Those are the identical transformation E and the group with a reflection $\{E, -E\}$. The fact of necessity of a rational masses ratio has been discovered and first lattices of discrete velocities for a mixture of gases have been constructed in paper[8], paper[9] is devoted to *spurious* invariants in discrete models.

Presenting velocity distribution functions $f_\alpha(v)$ by the expansions on Dirac delta-functions concentrated on the velocities of the discrete sets L_α,

$$f_\alpha(v) = \sum_{v_0 \in L_\alpha} \bar{f}(v_0) \delta(v - v_0), \tag{18}$$

and two-particle distribution functions of a pair of colliding particles of the sorts α and β by the expansions on Dirac delta-functions concentrated on bivectors from the invariant ensembles $L_{\alpha\beta}$,

$$f_{\alpha\beta}(\xi) = \sum_{\xi_0 \in L_{\alpha\beta}} \bar{f}_\alpha(v_0) \bar{f}_\beta(u_0) \delta(v - v_0) \delta(u - u_0), \tag{19}$$

we obtain an analog of the Boltzmann equation for discrete models for gas mixtures of M components in the following explicit form:

$$\frac{\partial \bar{f}_\alpha(v_0)}{\partial t} + v_0 \cdot \frac{\partial \bar{f}_\alpha(v_0)}{\partial r} = \sum_\beta \sum_{\{u_0 | (v_0, u_0) \in L_{\alpha\beta}\}} \frac{4\pi}{K} \sum_{k=1}^K b_{\alpha\beta}(v, \hat{R}_k) \left[\bar{f}_\alpha(v_0') \bar{f}_\beta(u_0') - \bar{f}_\alpha(v_0) \bar{f}_\beta(u_0) \right] \tag{20}$$

$$v_0 \in L_\alpha; \quad \alpha, \beta = 1,...M.$$

Where L_α are sets of discrete velocities $v_{0\alpha}$ for each mixture component depend only on the mass m_α; $L_{\alpha\beta}$ are sets of colliding pairs $(v_{0\alpha}, u_{0\beta})^T \in L_{\alpha\beta}$; M is number of mixture components. The definition $\{u_0 | (v_0, u_0) \in L_{\alpha\beta}\} \equiv L_\beta(v_0)$ means a set of all u_0 for which the pair (v_0, u_0) with the given v_0 is contained in the invariant ensemble $L_{\alpha\beta}$.

V. EXAMPLE OF DISCRETE VELOCITY MODEL CONSTRUCTED ON THE GROUP OF SYMMETRY OF A CUBE O_h

Rotation on angle B around axis $\boldsymbol{B}$ is given by formula:

$$\exp \hat{B} = \frac{1 - \cos B}{B^2} \hat{B}^2 + \frac{\sin B}{B} \hat{B} + 1 \tag{21}$$

where the operator (matrix) $\hat{B}$ is defined by the following expression:

$$\hat{B} \overset{df}{=} \boldsymbol{B} \times , \qquad \hat{B} v \overset{df}{=} \boldsymbol{B} \times v; \tag{22}$$

We will describe nontrivial rotations from the group O by vectors $\boldsymbol{B}_1,... \boldsymbol{B}_{23}$. Rotation on the zero angle (trivial rotation) is the unity transformation $\hat{E}$:

$$\boldsymbol{B}_0 = 0, \qquad \exp(\hat{B}_0) = \hat{E} \tag{23}$$

Symmetry of a cube includes three forth odder axises coming though centers of opposite faces of a cube (9 nontrivial rotations):

$$\{\boldsymbol{B}\}_{1,\ldots 9} = \pm\frac{\pi}{2}(1,0,0), \ \pm\frac{\pi}{2}(0,1,0), \ \pm\frac{\pi}{2}(0,0,1), \ \pi(1,0,0), \ \pi(0,1,0), \ \pi(0,0,1), \tag{24}$$

four third odder axises coming though opposite corners of the cube (8 nontrivial rotations):

$$\{\boldsymbol{B}\}_{10,\ldots 17} = \pm\frac{2\pi}{3}\left(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \tag{25}$$

and six second order axises coming through centers of opposite edges of the cub (6 nontrivial rotations):

$$\{\boldsymbol{B}\}_{18,\ldots 23} = \pi\left(0, \frac{\pm 1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \ \pi\left(\frac{1}{\sqrt{2}}, 0, \frac{\pm 1}{\sqrt{2}}\right), \ \pi\left(\frac{\pm 1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \tag{26}$$

One can obtain the group of all transformations of symmetry of a cube $\boldsymbol{O}_h$ by adding the inversion transformation $(-\hat{E})$ to the group $\boldsymbol{O}$. All 48 elements of the group can be written making use of definitions (21)— (26) as

$$\{\hat{R}\}_{O_h} = \left\{ \pm exp(\hat{B}_n), n = 0,1,\ldots,23 \right\}. \tag{27}$$

There are three Bravais lattices (cubic system), which are invariant under transformations from group $\boldsymbol{O}_h$. The first one is the simple cubic lattice Γ_c. Sites of that lattice are situated in corners of identical cubes:

$$w(k_1, k_2, k_3) = k_1 w_1 + k_2 w_2 + k_3 w_3, \qquad k_1, k_2 \ k_3 = 0, \pm 1, \pm 2 \ldots,$$

where vectors of elementary periods are

$$w_1 = (1,0,0), \ w_2 = (0,1,0), \ w_3 = (0,0,1) \ . \tag{28}$$

The body centered cubic lattice Γ_c^v is a lattice with sites, which are situated in vertices of cubs, and in its centers. Vectors of elementary periods of the lattice are three vectors from a center of the cube to any three vertices. One of the possible variants is:

$$w_1 = \left(\frac{-1}{2}, \frac{1}{2}, \frac{1}{2}\right), \ w_2 = \left(\frac{1}{2}, \frac{-1}{2}, \frac{1}{2}\right), \ w_3 = \left(\frac{1}{2}, \frac{1}{2}, \frac{-1}{2}\right). \tag{29}$$

A face-centered lattice Γ_c^f is a lattice with sites situated in corners of cubes and in centers of faces. Vectors of elementary periods of the lattice are three vectors from any corner of a cube to centers of three faces. One of the possible variants is:

$$w_1 = \left(0, \frac{1}{2}, \frac{1}{2}\right), \ w_2 = \left(\frac{1}{2}, 0, \frac{1}{2}\right), \ w_3 = \left(\frac{1}{2}, \frac{1}{2}, 0\right) \tag{30}$$

Here we will describe characteristics of discrete velocity models constructed on simple cubic lattice (28). Substituting elementary periods of this lattice (28) to (16) one will have a relevant discrete velocity model. For discrete velocities components we will have the following expressions:

$$v_x = \frac{m_0}{m_1} l_1 + k_1; \quad v_y = \frac{m_0}{m_1} l_2 + k_2; \quad v_z = \frac{m_0}{m_1} l_3 + k_3;$$

$$u_x = -\frac{m_0}{m_2} l_1 + k_1; \quad u_y = -\frac{m_0}{m_2} l_2 + k_2; \quad u_z = -\frac{m_0}{m_2} l_3 + k_3; \tag{31}$$

where $k_1, k_2, k_3 = 0, \pm 1, \ldots \pm k_m$; $l_1, l_2, l_3 = 0, \pm 1, \ldots \pm l_m$. It is seen from (31) that discrete velocity sets for components v_x, v_y, v_z are the same independent sets. To have a number of different discrete velocities less then number of collisions (bivectors from invariant ensemble) $\frac{m_1}{m_0}$ and $\frac{m_2}{m_0}$ should be rational numbers:

$$\frac{m_1}{m_0} = \frac{p_1}{q_1}; \quad \frac{m_2}{m_0} = \frac{p_2}{q_2}; \tag{32}$$

where p_1, p_2, q_3, q_4 are integer numbers and at list the following inequalities should be held

$$2l_m + 1 \geq p_1, p_2; \ 2k_m + 1 \geq q_1, q_2 \tag{33}$$

To obtain all different discrete velocities of particles with mass $m = \frac{p}{q}$ one should take indexes l and k in the range:

$$l = -l_m, \ldots (-l_m + p); \quad k = -k_m, \ldots k_m \quad \text{and} \quad l = (-l_m + p + 1), \ldots l_m; \quad k = (k_m - q), \ldots k_m$$

$$v_x = \frac{q}{p} l + k; \tag{34}$$

Total number of different discrete velocities v_x for particles with mass $m = \frac{p}{q}$ will be:

$$N_x\left(\frac{p}{q}\right) = p(2k_m + 1) + q(2l_m + 1) - pq, \qquad\qquad q \leq 2k_m + 1, \ p \leq 2l_m + 1 \tag{35}$$

$$N_x\left(\frac{p}{q}\right) = (2l_m + 1)(2k_m + 1), \qquad\qquad q > 2k_m + 1 \ \text{or} \ p > 2l_m + 1$$

For illustration we here consider the specific gas mixture consisting of $Ne(20)$, $NO(30)$ and $Ar(40)$ gases. The relevant discrete model has the following parameters:

$$l_m = k_m = 5; \quad m_0 = 20; \qquad p_1 = 1, \ q_1 = 1; \quad p_2 = 3, \ q_2 = 2; \quad p_3 = 2, \ q_3 = 1; \tag{36}$$

For one-dimensional case ($v_y = v_z = 0$) according to (35) the number of different velocities for each gas component of the mixture will be:

$$N\left(\frac{1}{1}\right) = 21, \quad N\left(\frac{3}{2}\right) = 49, \quad N\left(\frac{2}{1}\right) = 31 \tag{37}$$

The number of effective collisions (the number of bevectors from invariant ensemble with nonzero relative velocity, $l_1 \neq 0$) for each two gases of the mixture is

$$N_c = 2l_m \times (2k_m + 1) = 110 \tag{38}$$

and

$$N_c > \quad 2N\left(\frac{1}{1}\right), \quad N\left(\frac{1}{1}\right) + N\left(\frac{3}{2}\right), \quad N\left(\frac{1}{1}\right) + N\left(\frac{2}{1}\right), \quad 2N\left(\frac{3}{2}\right), \quad N\left(\frac{3}{2}\right) + N\left(\frac{2}{1}\right), \quad 2N\left(\frac{2}{1}\right) \tag{39}$$

$$110 > (2 \times 21 = 42), (21 + 49 = 70), (21 + 31 = 52), (2 \times 49 = 98), (49 + 31 = 80), (2 \times 31 = 62)$$

Due to inequalities (39) this discrete model do not has *spurious* invariants.

The two dimensional $(v_z = 0)$ model with $l_m = k_m = 3$ has the following characteristics:

$$l_m = k_m = 3; \quad m_0 = 20, \quad p_1 = 1, \ q_1 = 1; \quad p_2 = 3, \ q_2 = 2; \quad p_3 = 2, \ q_3 = 1; \tag{40}$$

$$N\left(\frac{1}{1}\right) = 169, \quad N\left(\frac{3}{2}\right) = 841, \quad N\left(\frac{2}{1}\right) = 361; \qquad N_c = \left[(2l_m + 1)^2 - 1\right] \times (2k_m + 1)^2 = 2352$$

And calculated in the same way there are characteristics of the full three dimensional discrete model:

$$l_m = k_m = 2, \quad m_0 = 20, \quad p_1 = 1, \ q_1 = 1; \quad p_2 = 3, \ q_2 = 2; \quad p_3 = 2, \ q_3 = 1; \tag{41}$$

$$N\left(\frac{1}{1}\right) = 729, \quad N\left(\frac{3}{2}\right) = 6859, \quad N\left(\frac{2}{1}\right) = 2197; \qquad N_c = \left[(2l_m + 1)^3 - 1\right] \times (2k_m + 1)^3 = 15500$$

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